Robot Self-Assembly as Adaptive Growth Process: Collective Selection of Seed Position and Self-Organizing Tree-Structures

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***Abstract*— Autonomous self-assembly allows to create structures and scaffolds on demand and automatically. The desired structure may be predetermined or alternatively it is the result of an artificial growth process that adapts to environmental features and to the intermediate structure itself. In a self-organizing and decentralized control approach the robots interact only locally and form the structure collectively. Designing a complete approach that allows the robot group to collectively decide on where to start the self-assembly, that adapts at runtime to environmental conditions, and that guarantees the structural stability, is challenging and does not yet exist. We present an approach to self-assembly inspired by diffusion-limited aggregation that generates an adaptive structure reacting to environmental conditions in an artificial growth process. During a preparatory stage the robots collectively decide where to start the self-assembly also depending on environmental conditions. In the actual self-assembly stage, the robots create tree-like structures that grow towards light. We report the results of robot self-assembly experiments with 50 Kilobots. Our results demonstrate how an adaptive growth process can be implemented in robots. We briefly describe our future work of how to extend the approach to a 3-d growth process and how robot self-assembly as an open-ended adaptive growth process opens up a multiplicity of future opportunities.**

I. Introduction

Self-assembly is a powerful tool in nature that operates on all scales be it on the level of molecules, cells, or organisms [1]. Trying to create similar capabilities in engineered systems is very challenging. Promising are observations of simple self-organized pattern formations, such as the Brazil nut effect, that can inspire concise approaches in robotics [2]. Recent results in robot self-assembly [3] show that self-organizing approaches easily scale to large robot groups (10^3 robots). Other approaches that operate on smaller robot groups have shown that self-assembled robots can adapt to challenging environments and perform better than single robots [4], [5], [6], [7]. A related approach is modular robotics that focuses on dynamic reconfigurations of assembled robot modules [8], [9], [10], [11], [12]. Similar ideas are investigated in the field of programmable matter [13], [14], [15] where large numbers of robot modules assemble and interact to form desired shapes and to react to external inputs. There are also approaches to self-assembly that focus on the design of passive elements. These elements are driven by an external force (e.g., vibrations) to passively self-assemble [16]. Approaches to self-assembly in robotics can be separated into works that focus on self-assembly of predetermined or anticipated structures [3], [5] and works that focus on adaptive growth processes where only certain qualities of the resulting structure are specified [20], [21]. A third dimension is added by categorizing whether aspects of self-repair are considered [22]. An often overlooked requirement of autonomous self-assembly is that for a fully autonomous approach the robots also have to decide where and triggered by whom they want to start the self-assembly process. A similar problem exists also in swarm construction [23] where the starting problem is not always considered explicitly.

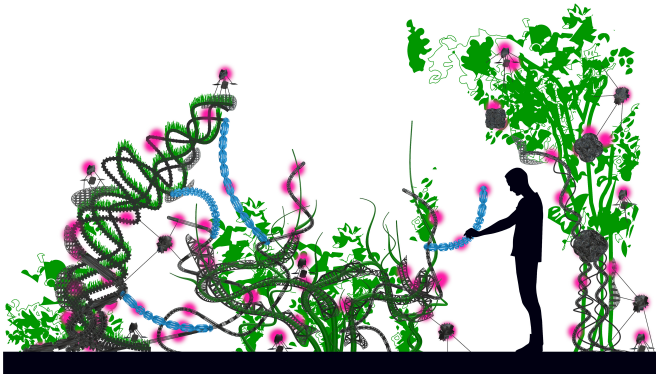

Fig. 1. Overall concept drawing, vision of the project *flora robotica* [17], [18], [19], societies of symbiotic bio-hybrids of robots and natural plants as social architectural artifacts, 3-d self-assembly with human interaction (by Mary Katherine Heinrich, [CITA] Centre for IT and Architecture, KADK)

In the following we present our approach to self-assembly as an adaptive growth processes. This research is done in the context of the EU-funded *flora robotica* research project [17], [18] that has the objective to create symbiotic relationships between a distributed robot system and natural plants (see Fig. 1). One of the main features is to control the growth of natural plants using robots and by making use of different tropisms of plants (e.g., phototropism: growth towards light). For that purpose we require the self-assembly of a robot-controlled scaffolding that grows in sync with the natural plant (within the project we will allow human interaction to self-assemble in three dimensions, in the following we focus on 2-d self-assembly without human interaction). Hence, it is useful to create an artificial growth process that has to mimic the natural growth process necessarily to a certain extent. We include the preparatory stage where the robots

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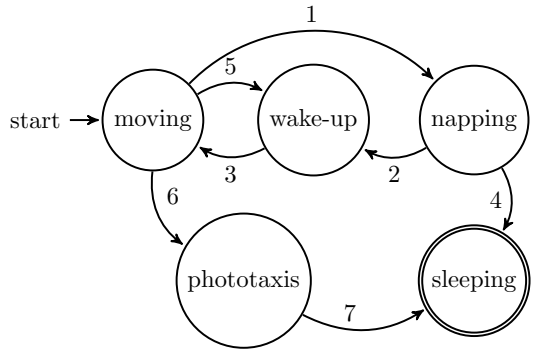


Fig. 2. Finite state machine of the robot controller.

have to collectively decide on where and triggered by whom they are going to start. We implement a collective decision-making process that selects a 'seed robot' that triggers the growth process. Instead of predetermining a certain concrete structure and shape, we predetermine reactions of the growth process to external stimuli (e.g., growth towards light) and qualitative features of the structure (e.g., branching ratio). The growth process itself is inspired by diffusion-limited aggregation (DLA), which is an aggregation process relying on random walk of particles that aggregate in a tree-like structure [24]. Although DLA is typically observed in chemical systems, investigations of networks with side branching revealed that trees and the vein structure of leaves have similar properties as structures grown by DLA [25]. Considerations about topologies that are cost-efficient, for example, that require minimal building material, are related to optimal transport networks [26].

In the following, we focus on a self-organized robot assembly forming trees that grow towards light (phototropism) and that have predetermined features. In our experiments we use the Kilobot [27] that was also used in the biggest robot self-assembly experiment (1024 robots) ever reported [3]. While we use a number of robots that is two magnitudes lower (50), we include the preparatory stage of collectively deciding on a seed and grow structures that were not predetermined and that adapt to the environment.

II. Control: BEECLUST and DLA

The robot self-assembly here is structured in two stages: preparatory stage and self-assembly stage. The robot controller is accordingly based on two different approaches. The preparatory stage is controlled by a modified version of the so-called 'BEECLUST' algorithm [28], [29], [30] that implements an aggregation process that reacts to environmental features. It is inspired by the behavior of young honeybees that aggregate at warm spots within the hive. Here we implement it such that it reacts to the light intensity field and selects a dark spot. We have defined it to aggregate at a dark spot for the practical reason that we want to grow the self-assembled structure towards the light later and need enough space within the arena. The preparatory stage is completed once a seed robot has been selected. The self-assembly stage is inspired by DLA [24] and grows a tree structure that we call 'DLA tree'.

In Fig. 2 we show the robot controller as finite state machine consisting of five states and seven transitions. Initially all robots are started in state *moving* and the final (accepting) state is *sleeping* which is the state of a robot aggregated in the DLA tree. Independent of their current state all robots (also those in states *napping* and *sleeping*) regularly broadcast a message via their IR emitter (every 32 ms). A receiving robot estimates the distance of the sending robot via the signal strength. We define that messages from senders at a distance of more than 5 cm are ignored. That scales the DLA tree and ensures that it fits into the robot arena. This message contains the robot's state, its depth (topological distance to the seed robot) within the tree (if applicable otherwise it is zero), the maximum depth of the tree (if applicable otherwise zero), and the maximum measured light intensity in the tree (if applicable otherwise zero). In state *moving* a robot performs a random walk, that is, it moves forward and makes a random turn on average every eight cycles. Once a robot in state *moving* receives a message, it either switches to state *napping* or *phototaxis* depending on the state of the other robot (part of the message). If the other robot is in state *napping* or *wake-up* then the considered robot switches to state *napping* (transition 1).

Similar to the BEECLUST algorithm, the robots measure the light only when they meet each other and we expect them to form clusters of *napping* robots. However, the stopping time in our method is fixed to three seconds which is different from the BEECLUST algorithm. The ambient light sensor generally gives values on the interval $a \in [0, 1023]$, but in our setting we have only $a \in [280, 1016]$. In addition, due to the position of the ambient light sensor on the robot it can give very different values at the same position but with different robot headings. After the waiting time has elapsed the robot switches to state *wake-up* (transition 2). In *wake-up* the robot does a random turn and moves forward at low speed for four seconds (low speed because it may be in the middle of a robot cluster and run into other robots). In state *wake-up* the robot ignores all incoming messages and especially does not switch back to *napping*. The idea is to allow robots to leave smaller clusters. After four seconds the robot switches to state *moving* (transition 3). Transitions 1, 2, 3 and the corresponding states implement an algorithm to select a seed robot (cf. leader selection).

Each robot looks for a light intensity value $a < 300$. The darkest spot has $a = 280$, hence, if a single robot would need to find it, it would search for long. However, the robots affect their neighbors' perception of the light distribution. A robot's shadow on the light sensor of a neighboring robot can largely influence the perceived light intensity. At the darker area the shadow is longer, denser robot clusters increase the probability of a shadowing effect, and as a result robots sense lower light intensity values. The reason is that a robot inside a cluster iterates over *napping* states followed by turns due to the *wake-up* process without leaving. Turning repeatedly with close-by robots increases the probability of getting a shadow on its light sensor. If the robot has

perceived a dark spot then it switches to state *sleeping* which is the state of robots that are aggregated in the DLA tree. Hence, a robot doing transition 4 becomes a seed robot and starts a new DLA tree. This is a probabilistic control approach and we cannot exclude the possibility that several robots become seed robots. That is acceptable because also several but few DLA trees still serve our purpose. A possible extension of this approach is to adapt to the environment by tuning light intensity thresholds dynamically using the robots' observations [31].

Robots in state *moving* that receive a message of a robot in state *sleeping* have the chance to join a DLA tree. However, we define that as a probabilistic behavior because we want to grow DLA trees of defined features. In particular, we want to grow trees with a defined branching ratio (i.e., widely ramified tree compared to the number of used robots) [32], [25]. Hence, we have to ensure that many robots join the DLA tree at the leaves (i.e., end-positions of the branches) and avoid that too many robots join the DLA tree at non-leaf positions. Every robot constantly sends a message to its local neighborhood which contains its state. The message also contains the robot's depth within the tree d, the maximum depth of the tree d_{max}, and the maximum measured ambient light intensity a_{max} in the tree. In the case of robots that are not in the *sleeping* state, only the state value of its message is used. The robot that is joining the DLA receives this message and calculates probability

$$P_d = Pr\left[X < \frac{d}{d_{\text{max}}}\right], \tag{1}$$

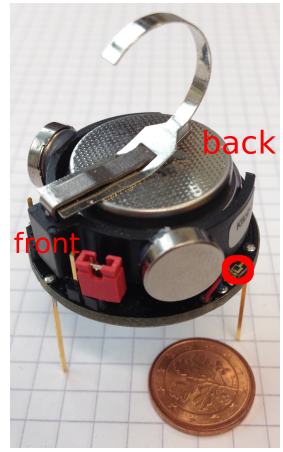
where X is a random variable with uniform distribution over interval $[0, 1]$. If $X < \frac{d}{d_{\text{max}}}$ then the robot joins the tree, otherwise it turns randomly and moves away (random numbers are generated by the Kilobot's hardware random number generator). P_d gives higher probability for tree depths close to the maximal depth of the tree (i.e., close to leaves). The second feature of the DLA tree, that we want to control, is the growth towards light. In plant science that is called phototropism [18]. The joining robot also measures the current ambient light intensity a and calculates probability

$$P_a = Pr\left[X < \frac{a}{a_{\text{max}}}\right], \tag{2}$$

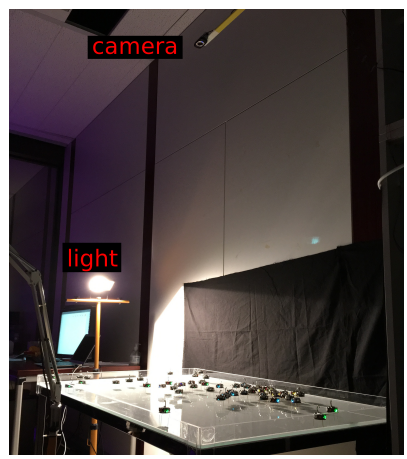
which gives a high probability if the measured ambient light at that position is close to the maximum measured ambient light in the tree. To implement a low branching ratio and to implement phototropism we define a probability P as product of the above probabilities:

$$P = P_d P_a. \tag{3}$$

We define the probability that the considered robot joins the DLA tree as P. If the robot does not join the DLA tree, then it uses transition 5 and switches to state *wake-up* (i.e., random turn and move forward at low speed). If the robot joins the DLA tree, then it uses transition 6 and switches to state *phototaxis*. In state *phototaxis* the robot moves towards the light for a short time by turning back and forth, permanently measuring the ambient light, and moving towards the light. Then it switches to state *sleeping* (transition 7) and stays aggregated in the DLA tree.



(a) Kilobot and 1-Eurocent coin, circle marks ambient light sensor



(b) experiment setup

Fig. 3. Kilobot and experiment setup.

III. Kilobot and experiment setup

We use the Kilobot [27] as shown in Fig. 3a which also indicates the positioning of the ambient light sensor on the robot (indicated by a circle). It has a diameter of about 3.3 cm and a battery that provides a few hours of energy autonomy. The Kilobot locomotes by stick-slip motion using three legs and a pair of vibration motors positioned at its sides. The Kilobot reaches a nominal speed of abut 1 cm/s and can turn on the spot with up to $\pi/4$ rad/s. It has an ambient light sensor is able to communicate infrared messages of 3 bytes up to a range of 10 cm to 20 cm depending on the reflection properties of the ground surface (in this work we limit the range to only 5cm). The robot arena is the surface of a glass table with dimensions 135 cm $\times$ 85 cm, see Fig. 3b. The light source is a halogen light with power consumption of 150 W positioned at the right-hand short side of the table at a distance of 45 cm and a height that

is 55 cm higher than the table's surface. This setup of the light combined with the position of the ambient light sensor on the Kilobot creates challenges when the ambient light needs to be measured at a certain position. If the robot's orientation in that moment causes the ambient light sensor to be at the shaded side of the robot, then it measures a low light intensity and vice versa. We have not implemented sophisticated methods, such as turning on the spot to scan the light intensity or to keep a history of recently measured light intensities, because we did not want to slow-down the self-assembly process or complicate the approach. Instead we accept the rather probabilistic success of measuring the ambient light and rely on the robustness of our approach to noisy measurements.

IV. Results

We have done 8 experiments with 50 robots. Initially the robots are approximately uniformly distributed in the arena and in state *moving*. An experiment is run for 60 minutes. A video of a complete experiment[1] and an accelerated video[2] (supplementary material) are provided online. The figures of these experiments and the accelerated video are also available[3] at Zenodo[4]. In Fig. 4 we give photos of the robot arena taken at the end of each experiment and indicate the resulting DLA tree. During the preparatory stage, the collective decision-making approach successfully selects a seed robot in the darker area of the arena as desired. In one of the experiments (experiment d, Fig. 4d) we observed two seed robots in all other experiments only one. Only one of the two seeds in experiment d was able to form a bigger DLA tree while the other seed only collects two additional robots.

The adaptive self-assembly process then is successfully forming trees with distinguishable bifurcations and branches that grow towards the light at the left-hand side of the arena. Between three and five robots are not yet aggregated after the experiment duration of 60 minutes in each of the experiments. Based on our experience the number of robots joining the DLA tree in the beginning is high but gets lower with lower number of available *moving* robots. That is expected because the number of robots that approach the tree is getting lower with decreasing number of *moving* robots.

To test the effectivity of the implemented phototropism (eq. 2), we define a measure to estimate the ratio of the DLA tree that was growing in the right direction. For that we define a triangle between the left-hand north corner of the arena, the left-hand south corner, and the seed robot. As a tolerance we increase the size of this triangle as shown in Fig. 5. We calculate the ratio of the footprints and parts of robots that are positioned on that triangle (we call that 'biomass') compared to the overall footprint of the whole robot group. In Fig. 5 we give processed images of the robots' end positions indicating the biomass on the target area in white. The percentages of biomass on the target area for experiments a to h are 26%, 82%, 57%, 28%, 38%, 30%, 44%, and 77% which gives an average of 47%. Experiment b has a percentage of 82% because the seed robot is positioned at the far right-hand side of the arena and as a consequence the target area is big. We consider the average percentage of 47% as satisfying. The light intensity mainly differs along the long side of the arena (left/right) and differences in the light intensity along the short side of the arena (north/south) cannot be measured by the Kilobot. Hence, it is possible that the DLA tree grows also in width along the short side of the arena. In Fig. 6 we give the evolution of the self-assembly over time for experiment b. The tree has relatively few and long branches as desired. Hence, our definition of the joining probability (eq. 3) seems efficient.

We have analyzed the trees using the branching ratio as defined by Horton [32], [25]. In his classification, a branch that ends with a leaf has order $i = 1$. When two first-order branches combine we get a second order branch ($i = 2$). When two second-order branches combine we get a third order branch ($i = 3$) and so on. When a second-order branch and a first-order branch combine we still have a second-order branch. The total number of ith order branches is N_i (a connected sequence of branches of the same order are counted as one). The branching ratio is defined as

$$R_N = \frac{N_i}{N_{i+1}}. \tag{4}$$

We have evaluated the branching ratio for the resulting trees (e.g., experiment b: $N_1 = 15$, $N_2 = 4$, $N_3 = 1$, $N_1/N_2 = 3.75$ and $N_2/N_3 = 4$; experiment c: $N_1 = 12$, $N_2 = 3$, $N_3 = 1$, $N_1/N_2 = 4$ and $N_2/N_3 = 3$). Summarizing over all eight experiments, we get means of $\overline{N}_1 = 14$ (standard deviation 2.27), $\overline{N}_2 = 3$ (0.756), and $\overline{N}_3 = 0.875$ (0.35). For the branching ratio we get two values $R_N^1 = \overline{N}_1/\overline{N}_2 \approx 4.67$ and $R_N^2 = \overline{N}_1/\overline{N}_2 \approx 3.43$. These are similar ratios as, for example, Horton [32] gets for rivers ($2.2 < R_N < 3.9$) and are below the statistics for DLA trees with $R_N \approx 5.15$ [25]. By changing the probability for a robot of joining the DLA tree (eq. 1) we could probably influence the branching ratio but that is left for future work.

V. Conclusions

We have presented a control algorithm for adaptive self-assembly that generates a self-organizing growth process. The growth is directed by growing towards light (phototropism) and we achieve a reasonable branching ratio with the resulting tree-structure even with noisy measurements of the ambient light. Our approach includes a preparatory stage at which the robots make a collective decision about where they start to grow the structure and who initiates it. We have reported robot experiments that show the successful growth of tree-structures towards light. Besides simple tasks such as growing a maximal amount of biomass into a certain region, we will also investigate more complex tasks in future research. For example, we will investigate the growth

[1] https://youtu.be/Vn5Vmh_YIoY

[2] https://youtu.be/jLfbs6X1tP0

[3] https://zenodo.org/record/58703

[4] see https://zenodo.org, Zenodo is developed by CERN under the EU FP7 OpenAIREplus (grant agreement no. 283595)

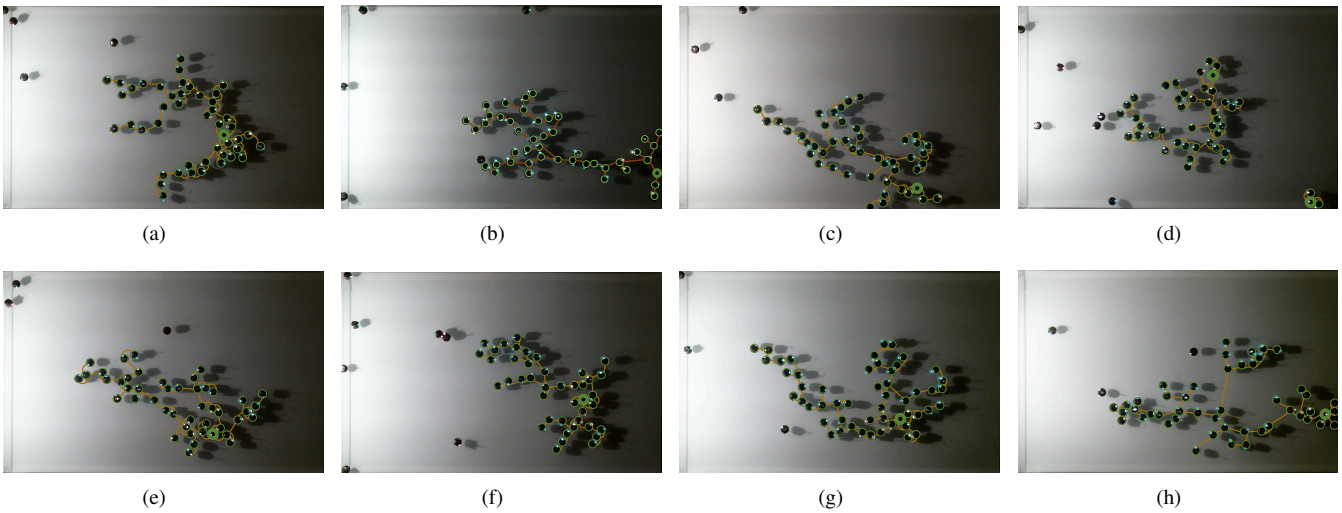


Fig. 4. Photos of robot end positions for each experiment. The light is positioned at the left-hand side. Between three and five robots are not yet aggregated in each of the experiments. DLA trees of the resulting arrangement of the robots are shown as well. The seed robots are highlighted with thicker circles around those robots. Note that in *d* there are two seed robots.

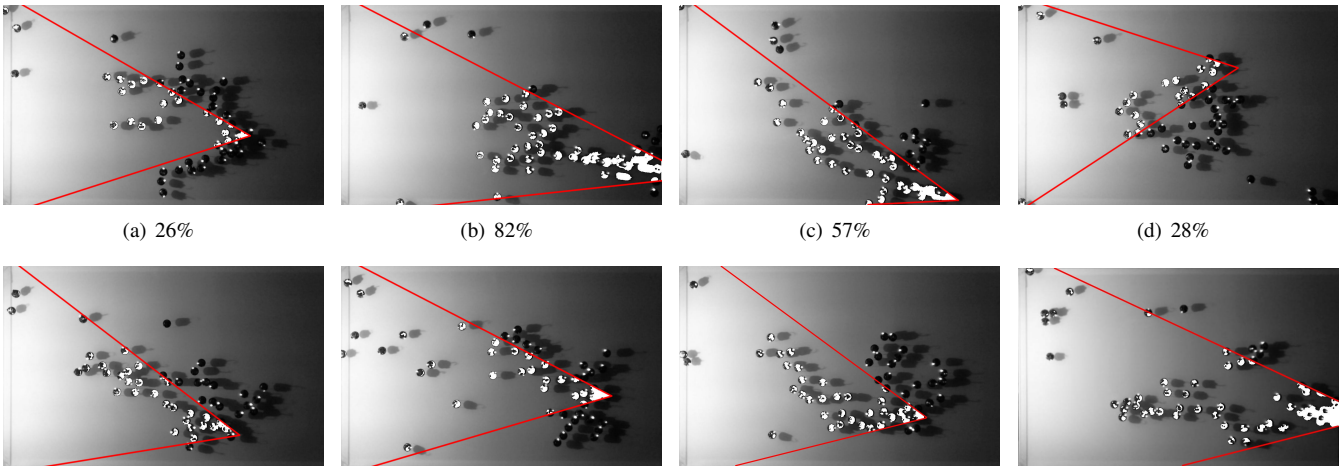


Fig. 5. End positions of robots with their 'biomass' on the target area (red triangle) indicated in white and given as percentage.

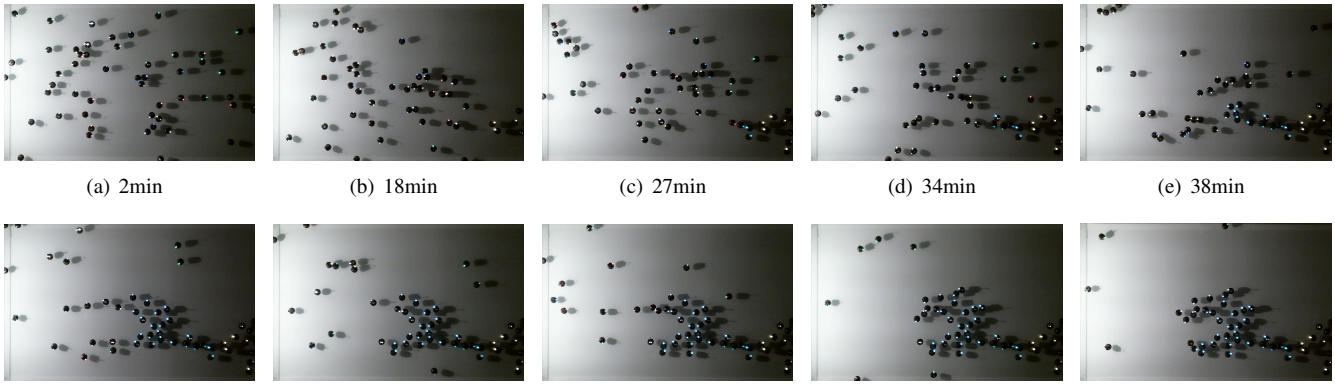


Fig. 6. DLA tree and self-assembly over time for experiment *b*. The RGB LED of the robots in the DLA indicates their depth in the tree as determined by themselves (yellow for depth smaller than 4, blue for depth between 4 and below 16, white for 16 and more).

of networks with desired features, such as balanced trees, determined edge lengths, and minimal route factor [33]. This research is done in the context of the EU-funded project *flora robotica* [17], [18] which requires the development of a robot-controlled scaffolding that parallels the growth process of natural plants. In future research, we plan to use the approach presented in this paper to grow three-dimensional tree-structures. For that purpose we currently develop dedicated robot hardware. Although the focus will switch from autonomous self-assembly to self-assembly with human interaction (robots autonomously determine the structure but request human interaction to attach additional robot components), we will be able to re-use the methods reported here. Within *flora robotica* we will use 3-d self-assembly as artificial growth in parallel to natural plants to guide their growth. Robots and plants will form a bio-hybrid with many applications, such as growing desired shapes and forms (e.g., architectural artifacts: walls, roofs, benches). The system is supposed to work in close interaction with human beings, to detect their requirements, and to interact with them. The methods for self-assembly reported in this paper will be key to achieve that.

References

[1] G. M. Whitesides and B. Grzybowski, "Self-assembly at all scales," *Science*, vol. 295, no. 5564, pp. 2418–2421, 2002.

[2] R. Groß, S. Magnenat, and F. Mondada, "Segregation in swarms of mobile robots based on the brazil nut effect," in *IEEE/RSJ International Conference on Intelligent Robots and Systems (IROS 2009)*. IEEE, 2009, pp. 4349–4356.

[3] M. Rubenstein, A. Cornejo, and R. Nagpal, "Programmable self-assembly in a thousand-robot swarm," *Science*, vol. 345, no. 6198, pp. 795–799, 2014. [Online]. Available: http://www.sciencemag.org/content/345/6198/795.abstract

[4] R. Groß, M. Bonani, F. Mondada, and M. Dorigo, "Autonomous self-assembly in swarm-bots," *Robotics, IEEE Transactions on*, vol. 22, no. 6, pp. 1115–1130, 2006.

[5] M. Dorigo, E. Tuci, V. Trianni, R. Groß, S. Nouyan, C. Ampatzis, T. H. Labella, R. O'Grady, M. Bonani, and F. Mondada, "SWARM-BOT: Design and implementation of colonies of self-assembling robots," in *Computational Intelligence: Principles and Practice*, G. Y. Yen and D. B. Fogel, Eds. Los Alamitos, CA: IEEE Press, 2006, pp. 103–135.

[6] R. O'Grady, R. Groß, A. L. Christensen, F. Mondada, M. Bonani, and M. Dorigo, "Performance benefits of self-assembly in a swarm-bot," in *IEEE/RSJ International Conference on Intelligent Robots and Systems (IROS)*, Oct 2007, pp. 2381–2387.

[7] W. Liu and A. F. T. Winfield, "Distributed autonomous morphogenesis in a self-assembling robotic system," in *Morphogenetic Engineering: Toward Programmable Complex Systems*, R. Doursat, H. Sayama, and O. Michel, Eds. Berlin, Heidelberg: Springer, 2012, pp. 89–113. [Online]. Available: http://dx.doi.org/10.1007/978-3-642-33902-8_4

[8] S. Murata, K. Kakomura, and H. Kurokawa, "Toward a scalable modular robotic system - navigation, docking, and integration of M-TRAN," *IEEE Robotics & Automation Magazine*, vol. 14, no. 4, pp. 56–63, 2008.

[9] H. Wei, Y. Cai, H. Li, D. Li, and T. Wang, "Sambot: A self-assembly modular robot for swarm robot," in *Robotics and Automation (ICRA), 2010 IEEE International Conference on*, May 2010, pp. 66–71.

[10] P. Levi and S. Kernbach, Eds., *Symbiotic Multi-Robot Organisms: Reliability, Adaptability, Evolution*. Springer, Feb. 2010.

[11] H. Hamann, J. Stradner, T. Schmickl, and K. Crailsheim, "A hormone-based controller for evolutionary multi-modular robotics: From single modules to gait learning," in *Proceedings of the IEEE Congress on Evolutionary Computation (CEC'10)*, 2010, pp. 244–251.

[12] K. Støy, "Using cellular automata and gradients to control self-reconfiguration," *Robotics and Autonomous Systems*, vol. 54, no. 2, pp. 135–141, 2006.

[13] T. Toffoli and N. Margolus, "Programmable matter: concepts and realization," *Physica D: Nonlinear Phenomena*, vol. 47, no. 1, pp. 263–272, 1991.

[14] S. C. Goldstein, J. D. Campbell, and T. C. Mowry, "Programmable matter," *Computer*, vol. 38, no. 6, pp. 99–101, 2005.

[15] M. A. McEvoy and N. Correll, "Materials that couple sensing, actuation, computation, and communication," *Science*, vol. 347, no. 6228, p. 1261689, 2015.

[16] E. Klavins, "Programmable self-assembly," *Control Systems, IEEE*, vol. 27, no. 4, pp. 43–56, 2007.

[17] *flora robotica*, "project website," 2016, http://www.florarobotica.eu.

[18] H. Hamann, M. Wahby, T. Schmickl, P. Zahadat, D. Hofstadler, K. Støy, S. Risi, A. Faina, F. Veenstra, S. Kernbach, I. Kuksin, O. Kernbach, P. Ayres, and P. Wojtaszek, "*flora robotica* – mixed societies of symbiotic robot-plant bio-hybrids," in *Proc. of IEEE Symposium on Computational Intelligence (IEEE SSCI 2015)*. IEEE, 2015, pp. 1102–1109.

[19] M. Wahby, D. N. Hofstadler, M. K. Heinrich, P. Zahadat, and H. Hamann, "An evolutionary robotics approach to the control of plant growth and motion: Modeling plants and crossing the reality gap," in *Tenth IEEE International Conference on Self-Adaptive and Self-Organizing Systems (SASO)*, 2016, (in press).

[20] J. Mullins, B. Meyer, and A. P. Hu, "Collective robot navigation using diffusion limited aggregation," in *Parallel Problem Solving from Nature-PPSN XII*. Springer, 2012, pp. 266–276.

[21] H. Hamann and H. Wörn, "Aggregating robots compute: An adaptive heuristic for the Euclidean Steiner tree problem," in *The tenth International Conference on Simulation of Adaptive Behavior (SAB'08)*, ser. LNAI, M. Asada, J. C. Hallam, J.-A. Meyer, and J. Tani, Eds., vol. 5040. Springer-Verlag, July 2008, pp. 447–456.

[22] M. Rubenstein and W.-M. Shen, "Scalable self-assembly and self-repair in a collective of robots," in *Proceedings of the IEEE/RSJ International Conference on Intelligent Robots and Systems (IROS)*, Oct. 2009.

[23] J. Werfel, K. Petersen, and R. Nagpal, "Designing collective behavior in a termite-inspired robot construction team," *Science*, vol. 343, no. 6172, pp. 754–758, 2014. [Online]. Available: http://www.sciencemag.org/content/343/6172/754.abstract

[24] J. T. A. Witten and L. M. Sander, "Diffusion-limited aggregation, a kinetic critical phenomenon," *Phys. Rev. Lett.*, vol. 19, pp. 1400–1403, 1981.

[25] D. Turcotte, J. Pelletier, and W. Newman, "Networks with side branching in biology," *Journal of Theoretical Biology*, vol. 193, no. 4, pp. 577–592, 1998.

[26] S. Bohn and M. O. Magnasco, "Structure, scaling, and phase transition in the optimal transport network," *Physical review letters*, vol. 98, no. 8, p. 088702, 2007.

[27] M. Rubenstein, C. Ahler, and R. Nagpal, "Kilobot: A low cost scalable robot system for collective behaviors," in *IEEE International Conference on Robotics and Automation (ICRA 2012)*, 2012, pp. 3293–3298.

[28] T. Schmickl and H. Hamann, "BEECLUST: A swarm algorithm derived from honeybees," in *Bio-inspired Computing and Communication Networks*, Y. Xiao, Ed. CRC Press, Mar. 2011.

[29] S. Kernbach, R. Thenius, O. Kornienko, and T. Schmickl, "Re-embodiment of honeybee aggregation behavior in an artificial micro-robotic swarm," *Adaptive Behavior*, vol. 17, pp. 237–259, 2009.

[30] H. Hamann, H. Wörn, K. Crailsheim, and T. Schmickl, "Spatial macroscopic models of a bio-inspired robotic swarm algorithm," in *IEEE/RSJ 2008 International Conference on Intelligent Robots and Systems (IROS'08)*. Los Alamitos, CA: IEEE Press, 2008, pp. 1415–1420.

[31] M. Wahby, A. Weinhold, and H. Hamann, "Revisiting BEECLUST: Aggregation of swarm robots with adaptiveness to different light settings," in *9th EAI International Conference on Bio-inspired Information and Communications Technologies (BICT 2015)*. ACM, 2016, pp. 272–279. [Online]. Available: http://dx.doi.org/10.4108/eai.3-12-2015.2262877

[32] R. E. Horton, "Erosional development of streams and their drainage basins; hydrophysical approach to quantitative morphology," *Geological society of America bulletin*, vol. 56, no. 3, pp. 275–370, 1945.

[33] M. T. Gastner and M. E. Newman, "Shape and efficiency in spatial distribution networks," *Journal of Statistical Mechanics: Theory and Experiment*, vol. 2006, no. 01, p. P01015, 2006.